Temporal Network Theory

An Introduction into Key Concepts

COS 597C: Theory of Natural Algorithms

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Topic Agenda

(Approx 30 min.)

- Types of Temporal Networks
- Network Representations
  - Lossless
  - Lossy
- Network Topologies
- Controllability, Motifs, and Other Patterns
- Temporal Network Manipulation
  - Shuffled Time Stamps
  - Random Link Shuffling
- Network Generators
  - Dynamic linking for static networks
  - Self-Exciting Point Processes
- Applications: Common Dynamic Systems

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https://tinyurl.com/holmetemporalnetworks

If you want to follow along, I will generally be presenting along the structure of the paper!

Types of Temporal Networks

Types of Temporal Networks

- It is natural to introduce and discuss temporal networks through the lens of static networks
- Temporal networks are based on a structure that links entities pairwise, and also encodes the time of interaction
- computer science pioneered temporal network theory - building off of ideas of Leslie Lamport
- Human Proximity Networks
  - Highest resolution data comes from RFID or infrared sensors
  - one study used people logged onto campus WiFi network, contact is recorded if 2 people are connected to same WIFI router at same time
- Patient Referral Networks
  - how many patients are transferred between wards of a hospital system
  - studying 295,108 Swedish patients over two years
- Sexual Contact Networks
  - sexual contacts are self-reported by Brazilian sex buyers at a web community.

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https://vimeo.com/31490438

Types of Temporal Networks

- Animal Proximity Networks
- Human Communication Networks
  - These register who called whom, or who sent text messages to whom
  - Boundary problem
- Collaboration Networks
- Citation Networks
  - all out-links of a node (paper) happen simultaneously (when the paper is published)
- Brain Networks
  - temporal correlations of the oxygen levels as measured by fMRI scanning.
- Distributed Computing
  - Advent of mobile computing creates an adversarial setting where many devices must be able to connect to a network in a decentralized manner
- Ecological networks
  - Food webs - antagonistic interactions between species

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Types of Temporal Networks

- Biological Networks
  - temporal reorganizations of the protein interaction network could predict and explain the development of breast cancer
  - Difficult to record reaction time in metabolic network

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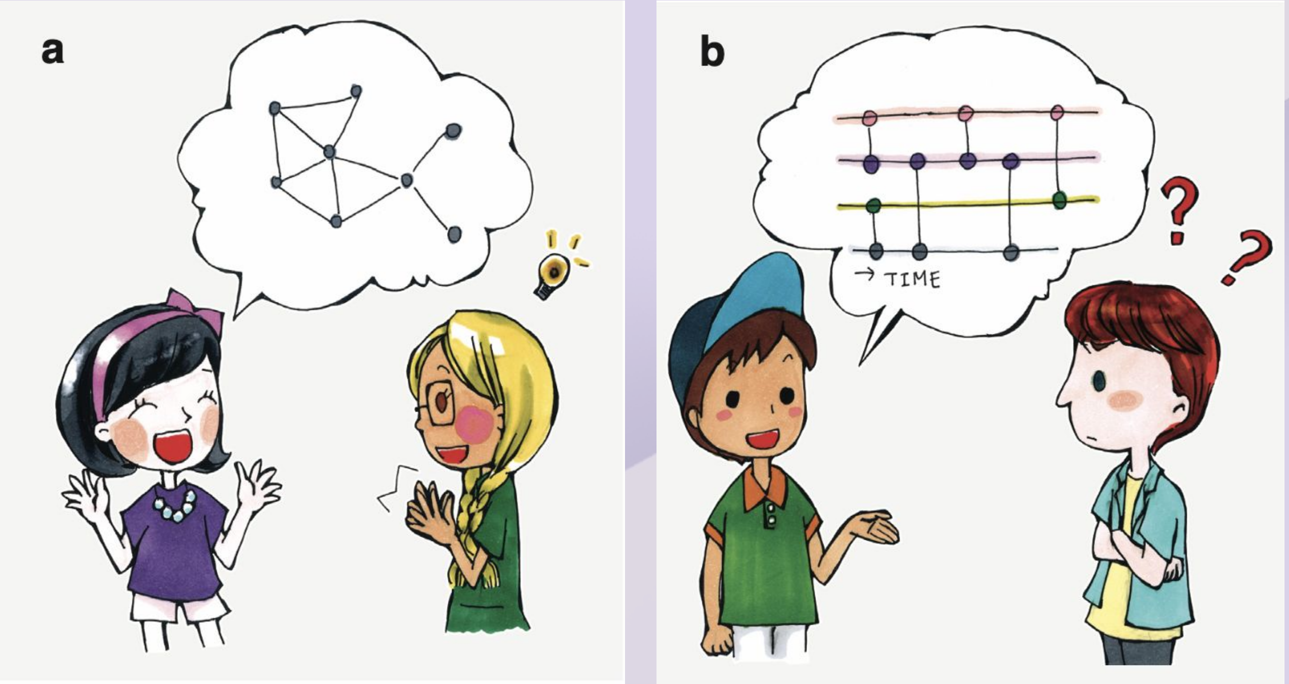
“We believe the readers of this paper are more imaginative that the author, so we will not try much harder”

Network Representations

How to Go About Representing Time Axis?



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Lossless Representations

- Contact Sequences
  - lists of contacts, i.e. the two node involved and the time of the interaction
  - good for computation, bad for visualization
- Graph sequences or multilayer networks
  - for any discrete time step, one can understand and characterize the network using static network theory
- Time-node graphs
- Time Series of contacts on a static graph
- Timelines of contacts
- Adjacency tensors
  - Just like a static network can be represented as a binary matrix, an adjacency matrix, a temporal network can be represented as a binary tensor
- Since the dynamic system of interest may not be able to operate within the graph of a time step, the adjacency tensor cannot function like an unnormalized Markov transition matrix.

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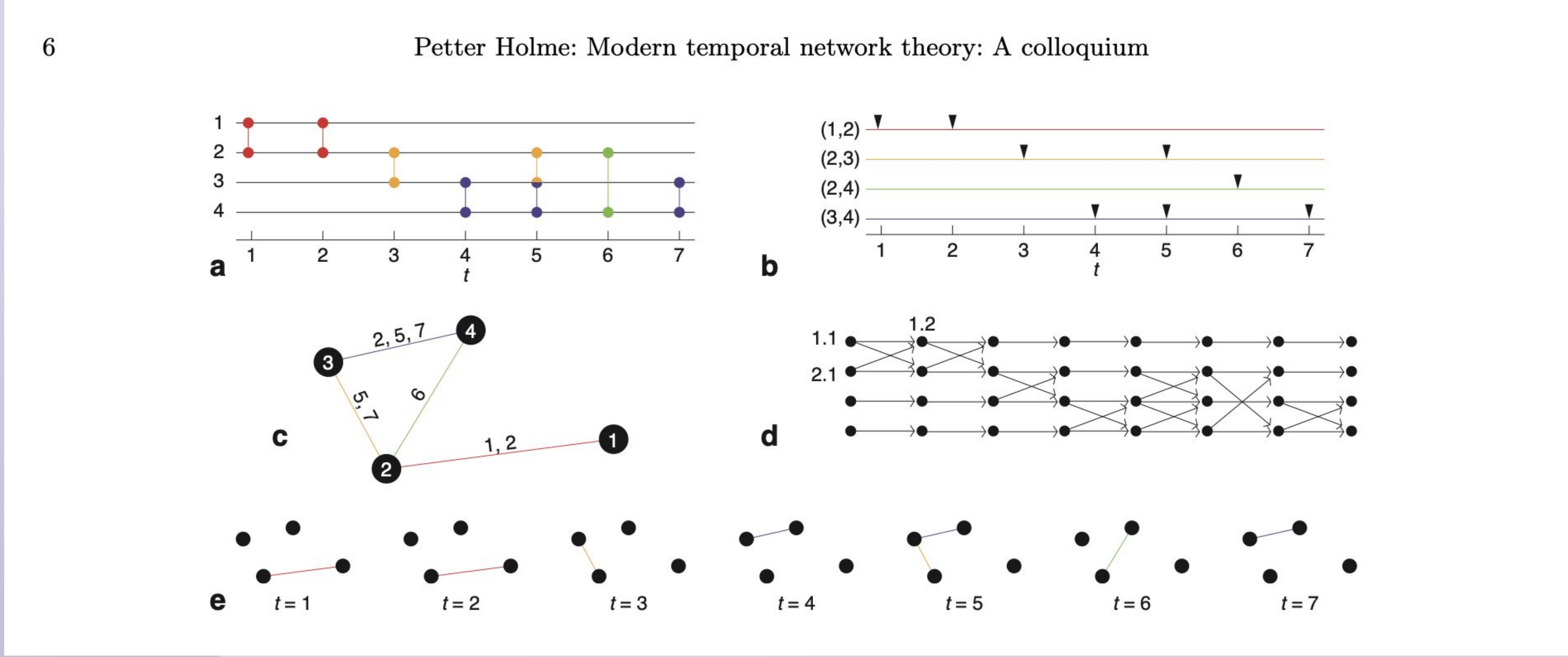
ACTIVITY 1 -

How many distinct, underlying temporal networks are represented in the 5 graphs below? (Anywhere from [1, 5])



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ACTIVITY 1

6 Petter Holme: Modern temporal network theory: A colloquium

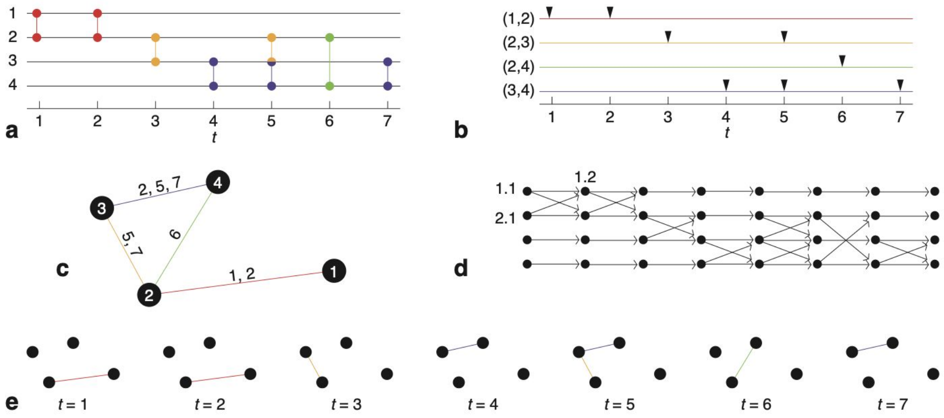

Fig. 2. The figure illustrates five representations of the same temporal network (of the contact sequence type). Panel (a) shows a node-centric time line, where a horizontal line represents a contact between two connected individual at the time given by the x-axis. Panel (b) shows a time line of the contacts focusing on links (pairs of individuals). Panel (c) shows a time-stamp decorated, aggregated graph. The numbers of the links denotes the contacts between the nodes. Panel (d) shows a time-node graph (where one assumes spreading cannot occur across more than one contact per time step). Three of the 32 time nodes are labeled. Panel (e) shows a graph sequence representation.

Lossy Representations

- Weighted Graphs
  - construct an link-weighted graph where the weight gives a count of the number of contacts between two nodes
- Reachability and influence graphs
  - directed, unweighted graph that links i to j if there is a time-respecting path from i to j
  - Can be less useful early in sampling period:
    - Can have 30- 100% of the maximum number N (N − 1) of directed links
- Time-Window Graphs
  - include all the links present in a time window
  - The very simplest way would be to take this time window as the entire sampling time
- Concurrency graphs
  - defines links in concurrency graphs as pairs of nodes having contacts both before T_start and after T_stop

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Lossy Representations

- Difference Graphs
  - Construct between two consecutive time steps by links that changed between time steps
- Memory Networks
  - It assumes there is a recorded walk process on a set of nodes, like flight passengers on multi-hop itineraries.

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Lossy Representation

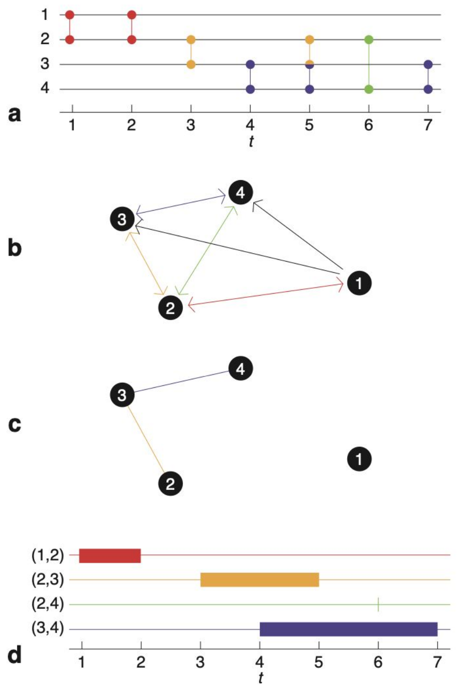

Fig. 3. Panel (a) shows an example contact sequence (the same as in Fig. 2). Panel (b) is the derived reachability graph at $t = 0$. Panel (c) shows the time-windowed static graph from contacts between $t = 3$ and $t = 5$. Panel (d) illustrates a link-turnover graph where there is a link between two nodes if they had a contact before and will have one again.

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Weighing Scheme Example

$$\sum_i e^{-t_i/\tau} \tag{1}$$

(where t_i is the time between the beginning of the spreading and the i'th contact, and τ is a parameter that should match the time scale of the dynamic process).

Suggestion: Static network defined by sum of exponentially decaying weights

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Network Topologies

Network Topology

- For static networks, there was a huge effort both to measure degree distribution and to model their emergence
    - No such ubiquitous structures for temporal networks yet
- Burstiness and inter-event time statistics are important to temporal networks
- Inter-event time distribution
    - In a time series of events this is the frequency distribution of the time between the events
- Generalized Distances
    - define the temporal distance τ (i, j, t) as the earliest time to reach from i to j on a time-respecting path starting at t
- Temporal Coverage Centrality
    - fraction of node pairs (j, j ') s.t passing i would not increase the time to reach from j to j '

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Centrality Measures

$$C_C(i) = \left[\frac{1}{N-1} \sum_{j \neq i} d(i,j) \right]^{-1} \tag{2}$$

where $d(i,j)$ is the graph distance. This can straightforwardly be extended to temporal networks as [315, 208, 23, 139]

$$C_C(i,t) = \left[\frac{1}{N-1} \sum_{j \neq i} \tau(i,j,t) \right]^{-1} . \tag{3}$$

A problem with closeness centrality in static networks which becomes much aggravated for temporal networks is that there might not be any path between i and some of the other nodes. A workaround is to average the inverse distance rather than inverting the average distance [213]

$$C_C(i,t) = \frac{1}{N-1} \sum_{j \neq i} \frac{1}{\tau(i,j,t)} . \tag{4}$$

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Controllability, Motifs, and Other Patterns

Controllability & Motifs

- Assumes a system with in and output terminals connected into a network (think of Neural Nets)
  - output from one node is proportional to (or at least a function of) the input
- there can be a phase transition in the time scale of the dynamics between a state where the network is controllable by a a vanishing faction of nodes

- patterns to look for in temporal networks are links/subgraphs that do not change as much as others
- Motifs
  - small subgraph that is overrepresented in a graph compared to in a null model
  - temporal patterns involving the links of a triangle within a short time period are common and important for spreading phenomena
- Network Communities
  - subgraphs that are densely connected within and sparsely connected to other communities during a time window.

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Mesoscale Structures: Community Detection

In other words, at time t one would first run the community detection algorithm on the static network to decompose the nodes V into communities $c_{1,t}, \ldots, c_{n(t),t}$ so that $\bigcup_i c_{i,t} = V$. The next step is to merge communities at t with overlapping communities at $t-1$. There are many ideas in the literature how to do that. Ref. [85] maps the indices of t to the indices of $t-1$ so that the sum of mismatching indices is minimized. This approach

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Mesoscale Structures: Community Detection

$$1 - \frac{|c_t \cap c_{t'}|}{|c_t \cup c_{t'}|} e^{-\gamma(|t-t'|-1)},$$

where c_t and $c_{t'}$ are two communities at two different times t and t'. The idea of the exponential factor is to de-

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Temporal Network Manipulation

Randomization Techniques

- **With randomization, one can see how much faster or slower the spreading becomes because of the specific network structure isolated from the temporal structure**
- **study effects of correlations in the real data set without having to make an exhaustive list of the correlations (think data augmentation)**
- **Shuffled Time Stamps**
  - **causes burstiness to decrease (clusters in points of time)**
- **Random Link Shuffling**
  - **This randomization procedure destroys all network topological structures except the degree sequence of the original graph (of accumulated contacts)**
- **Time Reversal**
  - **Time runs backwards**
- **best methodology for randomization is be to build a sequence of gradually more random ensembles**

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Shuffled Time Stamps

1. Iterate through the contacts. Let i be the current contact.
2. Take a random row j.
3. Swap the time stamps of rows i and j.

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Results of Randomization on Network

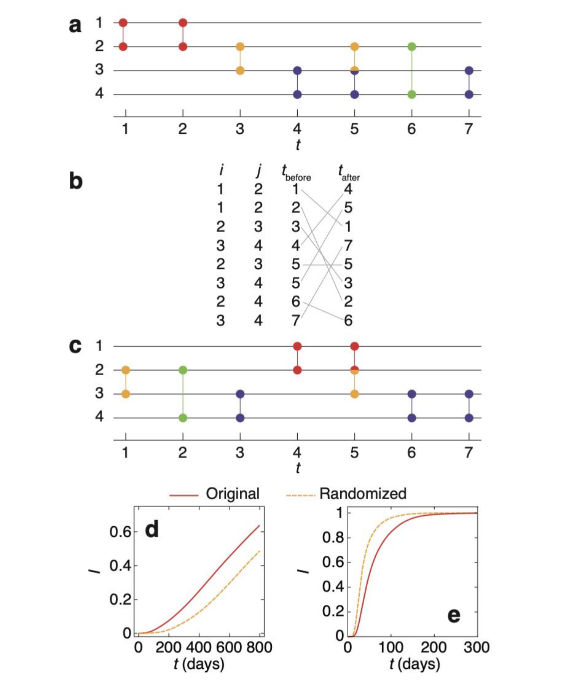

Fig. 4. Illustrating the shuffled-time-stamps scheme to randomize temporal networks. Panel (a) shows the original network from Fig. 2. Panel (b) shows how the randomization scheme operates on a contact-list representation of the data. In panel (c) we see the result as a nodal time-line plot. In panel (d), we see the effect of this type of randomization for susceptible-infectious (SI) spreading with 100% infection rate. The plot shows the average number of infectious nodes I as a function of time since the beginning of the data set t. The data comes from Ref. [247] and one can conclude that the order of event (that is destroyed by the randomization) speeds up the spreading. In panel (e), we see a corresponding plot for mobile phone data from Ref. [132] where the conclusion is the opposite—spreading is slowed down, in the data, by the structure of the order of the contacts.

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Random Link Shuffling

1. Pick an link (i, j) (sequentially) from the list of links.
2. Pick another link (i', j') randomly.
3. With equal probability replace (i, j) and (i', j') by (i, j') and (i', j), or by (i, i') and (j, j').
4. If the move in the previous step created a self-link or multiple link, then undo it and start over from step 2.
5. Go to step 1.

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Temporal Network Generators

Temporal Network Generators

- Face-to-face interactions
  - two-dimensional random walk model where there the chance of walking closer to a node i is proportional to an increasing attractiveness a_i
- Self-Exciting Point Processes
  - generalization of Poisson process s.t occurrence of event makes subsequent events in time/space more likely
  - a node forms and break links based on a Bernoulli process with memory

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Introducing Link Dynamics

1. Construct a simple graph by first generating a multi-graph by the configuration model [207], then removing duplicated links and self-links.
2. For every link, generate an active interval (when contacts can happen) from some distribution. Ref. [103] uses a truncated power-law for the duration of the active interval, and a uniformly random starting time within a sampling time frame.
3. Generate a sequence of contact times following some (bursty or not) interevent time distribution.
4. Wrap the contact time sequence onto the active intervals of the links. In other words, first rescale the sequence to the total time of the active intervals, then cut it in the same durations as the active intervals, and assign it to them.

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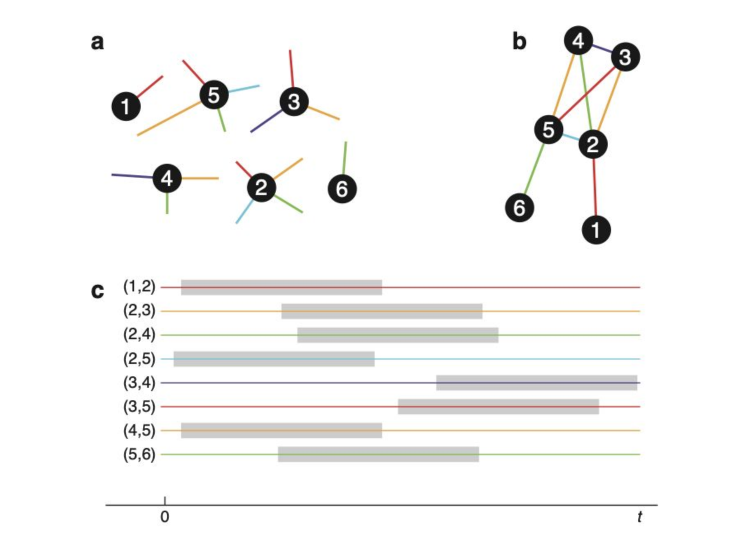
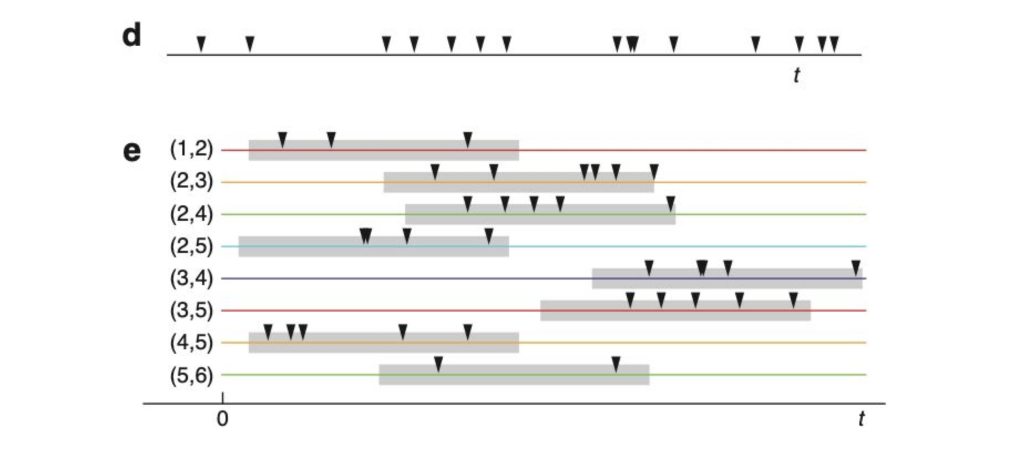
Introducing Link Dynamics

Fig. 5. Illustrating a simple generative model for temporal networks, used in Ref. [103] and (almost) in Ref. [243]. First one generates a static network (technically a multigraph) from the configuration model by (a) drawing degrees from a probability distribution and (b) matching them up in random pairs. Then one generates active intervals for the links (also randomly, in this case all links being active the same duration), (c). Finally one generates a time series of interevent times from a probability distribution (d) and rescales it and matches it to the active intervals.





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Activity Driven Network Generator

1. Increase the time counter to t and let G_t be empty.
2. For every node i, make it active with a probability $a_i \Delta t$. Connect i to m other randomly chosen distinct nodes (active or not). Ref. [223] uses a truncated power-law distribution for a_i.

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2D Random Walk

Starnini et al. [271] developed a model of temporal face-to-face networks. This is naturally a spatiotemporal network. Technically, their model is a two-dimensional random walk model where there the chance of walking closer to a node i is proportional to an increasing attractiveness a_i. The more attracted a walker is to its neighbors, the slower its walk becomes. Finally, they also model the agents as having active and inactive periods that they transfer between with the same probability every time step. The authors

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Self-Exciting Process Generator Example

$$v + \sum_{i:t_i \leq t} \phi(t - t_i) \tag{6}$$

where ϕ is an exponentially decreasing memory kernel (zero for negative arguments to respect causality) and v is a basal event rate. Even with an exponentially decreasing

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Applications: Common Dynamic Systems

Temporal Network for Dynamic System

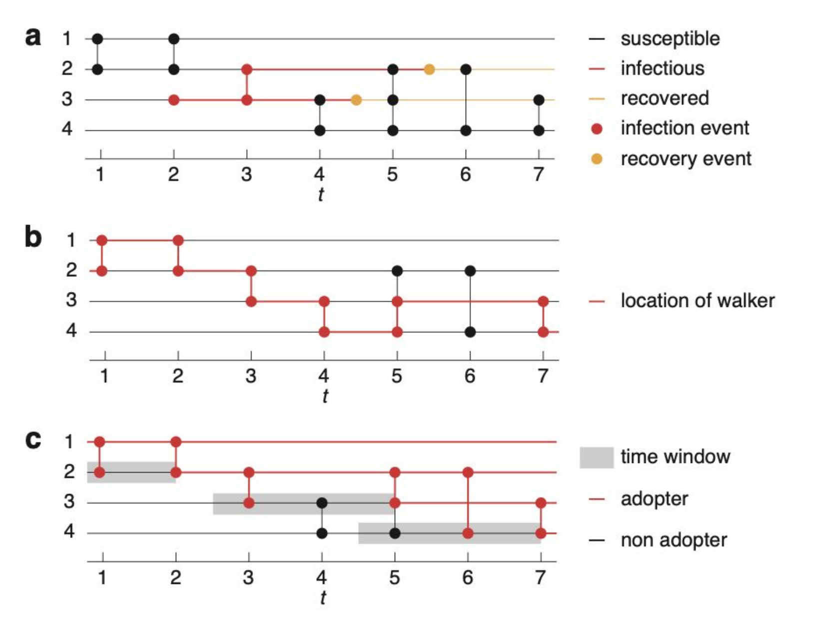

Fig. 6. Illustration of three dynamical systems on temporal networks. Panel (a) shows a susceptible-infectious-recovered model with a disease duration of 2.5 time steps. The outbreak starts at node 3 at time 2 and reaches one other node. There is a potentially contagious event between nodes 3 and 4 at time 4, but in this example chance made it not contagious. Panel (b) shows a greedy walk starting at node 2 at the beginning of the sampling period. A greedy walk follows every contact away from the node where the walker is. Panel (c) illustrates one of the temporal threshold models studied in Ref. [127]. A node becomes adopter if it is exposed to (i.e. in contact with an adopter) more than twice within a backward time window of 2.5 time units.

Applications of Dynamic Systems

- Walks
  - Usually the decision process has a random component, making it a random walk
- Epidemic Models
  - One interesting version of this is the vaccination problem—imagine you can immunize (or in other ways lower the impact with respect to spreading) a fraction f of a population,
  - Then how would you chose them?
- Opinion and information spreading
  - These assume that an individual adopts an idea when the exposure is over a threshold
  - older influence matters <<< newer, so one has to decide how to down- weigh older contacts
  - Counting contacts only within a moving window
- Other Potential Applications:
  - Percolation, Synchronization in brain networks, Evolutionary games / game theory

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